What Is the Best Primel Prime to Guess First?

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Abstract

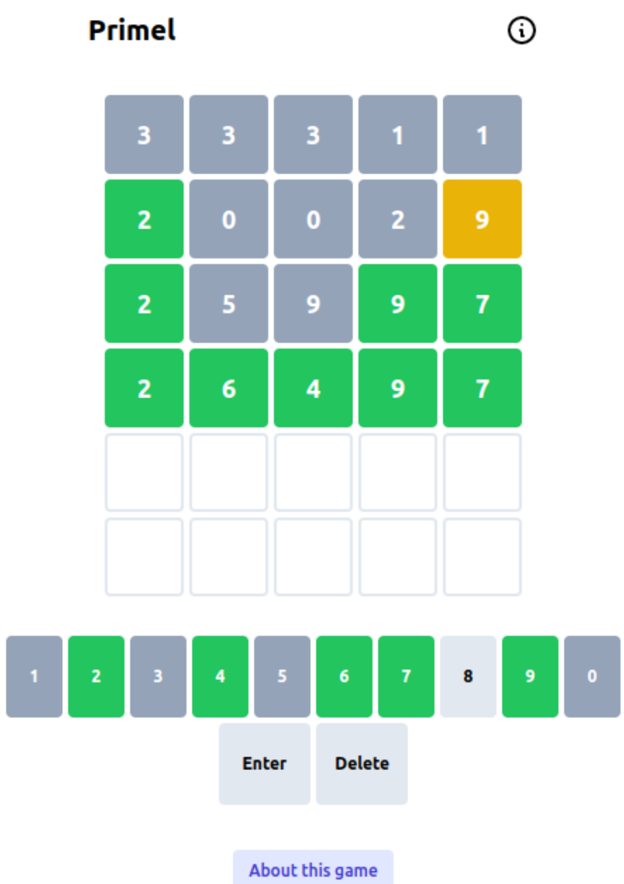
Primel is a variation on the popular word game, Wordle. In Primel, a person must repeatedly try to guess a five-digit prime number, receiving feedback from each unsuccessful guess, including what digits are in the correct spot, what digits are not in the correct spot, and what digits are not in the number at all. Since information is gleaned from each guess that may inform the subsequent guess, one's initial guess can make a big difference in how quickly one can guess the true number. In this paper, I investigate what the best initial guesses are. I additionally discuss variations of the game, strategy variations, and why some initial guesses are better than others.

Introduction

The recently popular word game, Wordle, has triggered a variety of game strategy research. (Anderson and Meyer 2022, de Silva 2022, Short 2022) This research has focused on the best starting words and the best overall strategy for making the correct guess quickest. The less-popular spin-off of Wordle, Primel, has not yet been investigated as thoroughly. In this paper I provide an initial analysis of Primel, including simple strategies, best initial Primel primes to guess, and variations to the game.

What is Primel?

Primel is a Prime number guessing game. Players attempt to guess a five-digit Prime number, receiving feedback from each unsuccessful guess. The feedback received includes information on which digits are in the correct spot, what digits are in the number but not in the correct spot, and what digits are not in the number at all. Players have six attempts.[1]



[1] It should be noted that the following work on Primel is for an additional variation of Primel where one is allowed up to 10 guesses.

Lazy Strategy

I first consider a single iterative strategy. I call this the *Lazy Strategy*, as it involves essentially random guessing after one pre-defined guess. While this is not the theoretically optimal strategy, this strategy is very easy to learn and implement, as unlike other strategies it does not require the player to make complex calculations, but rather just requires that they memorize a single number and then guess randomly based on the information provided.

1. Guess a pre-defined number
2. If the guess isn't correct, find all possible options for the Prime number given the information gleaned from the last guess/all previous guesses
3. Guess one of those options randomly
4. Repeat steps 2 through 4 until the guess is correct.

Two-digit Primel

I first consider this iterative strategy for two-digit Primel, since it's computationally simpler. These particular results are based on the archived script, with each prime being tested as an initial guess approximately 200,000 times.

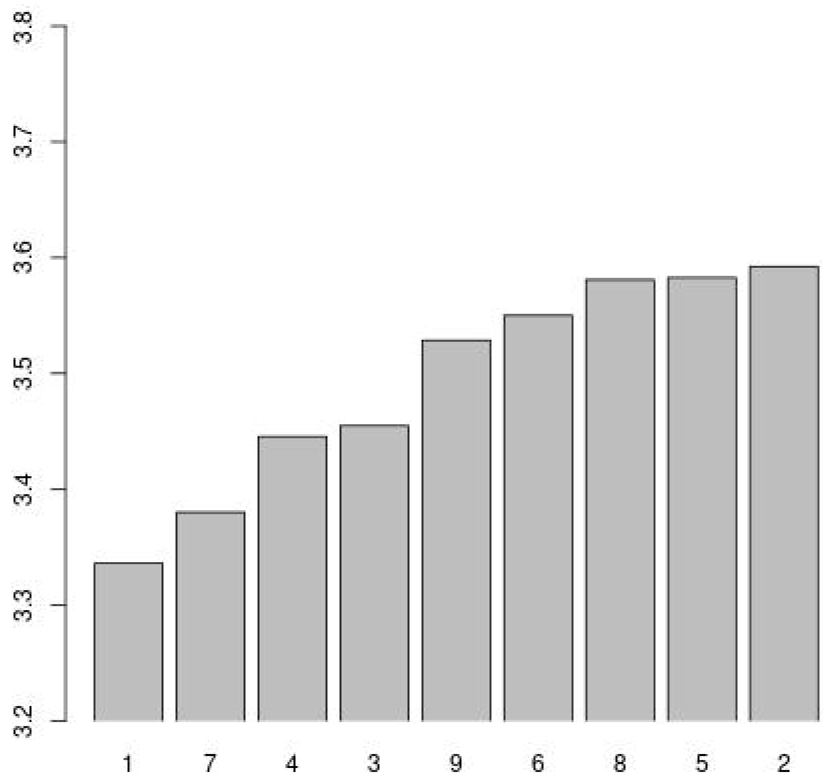
First guess	Rounds until correct guess
17	3.155358
71	3.20928
13	3.297963
31	3.354459
37	3.380309
41	3.384105
73	3.399043
47	3.418694
19	3.424679
97	3.442538
79	3.467077
61	3.528777
43	3.535725
53	3.551548
83	3.555717
23	3.567701
67	3.571386
89	3.605718
59	3.613944
29	3.617004
11	3.671746

The best initial guess is clear- 17. It's mostly obvious why this is a good initial guess too, 17 and 71 are both primes, so even if 17 is wrong as an initial guess, it may directly lead one to 71. It's also very likely that atleast one digit will be correct if one guesses 17, since a 1 or 7 is present in 12 of the 20 other 2-digit primes, with 7 of those exact matching on a 1 as the first digit or a 7 as the second digit.

The worst guess is clear too- 11. 11 is obviously a bad guess because it has only one unique digit, thus reducing the likelihood of a correct digit being guessed. What's somewhat surprising, is 11 is only a slightly worse guess than 29.

What are the most valuable digits?

The following table displays the average number of guesses to correct guess for each digit as it's found in each Prime number.[2]



Slightly-less Lazy Strategy

I now consider a second modified strategy. I call this the *Slightly-less Lazy Strategy*, as it involves random guessing after one to two pre-defined guesses.

1. Guess a pre-defined number
2. If no digits of the guess are correct, guess a pre-defined second guess. If one or more of the guesses are correct, skip to step 3.
3. If the guess isn't correct, find all possible options for the Prime number given the information gleaned from the last guess/all previous guesses
4. Guess one of those options randomly
5. Repeat steps 3 through 5 until the guess is correct.

[2] 11 is excluded.

We test all possible combinations of first and second guesses using the past strategy. The following combinations yield the best overall number of rounds until correct guess.[3]

First guess-Second guess	Rounds until correct guess
71-23	3.301887
37-61	3.314286
37-41	3.320388
13-79	3.321739
31-89	3.324561
17-53	3.33
23-17	3.330097
53-17	3.345794
31-97	3.35
13-29	3.352273

The following yield the worst overall number of rounds until correct guess.

First guess-Second guess	Rounds until correct guess
59-23	4.237113
67-89	4.149123
79-11	4.140187
89-11	4.13
89-23	4.12931
29-11	4.125
59-61	4.12381
59-11	4.08046
11-23	4.075472
29-83	4.046512

Interestingly, the second guess in the best-performing combination is the same as the second guess in the worst-performing combination. This interesting finding highlights an important property of guess combinations- a guess's value can be highly variable depending on other guesses that have been made. Certain guesses pair better with some more than others.

[3] This excludes cases where one or more digits matched on the first guess.

Three-digit Primel

Next, I return to the original *Lazy Strategy*, to examine three-digit Primel. Due to a greater number of options, I test all options a smaller number of times, approximately 14,000 times each.

Here are the best 10 options obtained:

First guess	Rounds until correct guess
349	3.694085
149	3.69507
641	3.697183
491	3.69838
463	3.700563
439	3.703099
643	3.707817
691	3.708732
461	3.710915
613	3.711338

Here are the worst ten options:

First guess	Rounds until correct guess
101	4.177042
557	4.170423
227	4.152324
881	4.144577
887	4.142887
229	4.141268
929	4.135352
811	4.13169
661	4.128451
223	4.126972

Interestingly, the worst-performing three-digit prime has similar properties to the worst-performing two digit prime. The worst three-digit prime, 101, is 10^2 +1 and is the lowest three-digit prime, while the worst two-digit prime, 11, is 10^1 + 1 and is the lowest two-digit prime.

Four-digit Primel

Next, again applying the original Lazy Strategy, I explore four-digit Primel. Due to an even greater number of options, I test all options an even smaller number of times, approximately 1,000 times each.

Here are the best 10 options obtained:

First guess	Rounds until correct guess
8263	3.74717
4657	3.772642
5869	3.772642
6427	3.775472
4759	3.779245
4261	3.782075
8623	3.783019
8123	3.783962
8369	3.783962
2459	3.788679

Here are the worst 10 options obtained:

First guess	Rounds until correct guess
1117	4.471698
3733	4.445283
5557	4.438679
7177	4.428302
1171	4.424528
1777	4.424528
1181	4.423585
9199	4.418868
7333	4.412264
2221	4.40566

Five-Digit Primel

Finally, applying the original Lazy Strategy, I investigate the real Primel: five-digit Primel. The approach for identifying best and worst guesses for five-digit Primel is slightly different than other game variations. Due to the large number of prime options in five-digit Primel, I first tested all numbers a smaller number of times. Based on that smaller number of simulations, I then identified the 50 best and 50 worst guesses. I then tested the 50 best and 50 worst guesses a larger number of times, to identify the very best and very worst initial guesses. The following two tables present the results of those simulations on the 50 best and 50 worst initial guesses.

First guess	Rounds until correct guess
82469	3.909962
42863	3.910918
56249	3.911074
56843	3.911325
84659	3.911552
24683	3.9116
54629	3.911827
46589	3.912641
82463	3.912712
26459	3.913035
85469	3.913681
84263	3.914076
62483	3.914853
48623	3.914877
84653	3.915152
68521	3.915881
56489	3.916097
82567	3.916718
85621	3.917675
28463	3.91789
62581	3.91911
28541	3.924659
28549	3.926477
62851	3.926919
84521	3.928797
52069	3.933222
50261	3.933389
20543	3.934621
20849	3.934621
46507	3.93663
24061	3.936714
50647	3.939189
40583	3.939751
20857	3.944152
56237	3.951985
57269	3.953073
45361	3.953145
24763	3.95336
61543	3.953671
46831	3.953923
45691	3.9547
26357	3.954772
41863	3.955226
26879	3.957044
89653	3.958276
62981	3.960309
46381	3.960536
68947	3.961899
51287	3.961995
64951	3.962413

First guess	Rounds until correct guess
33377	4.677386
77171	4.674348
33331	4.673655
77977	4.673021
71711	4.672315
71171	4.672279
13331	4.672255
33773	4.672088
99191	4.671873
37337	4.670761
77711	4.670294
13313	4.669888
11117	4.66978
17117	4.669517
79999	4.669206
99119	4.668668
77773	4.667711
77377	4.667627
11717	4.66752
77797	4.667233
79979	4.667089
33311	4.666563
19919	4.666503
77999	4.666396
11171	4.665953
71777	4.665008
31333	4.664925
11119	4.663765
22229	4.663585
33113	4.663179
97777	4.662975
11113	4.656506
78887	4.656135
15551	4.649438
22727	4.648182
22277	4.647345
72227	4.646986
83833	4.629933
38833	4.628857
78787	4.627493
11551	4.625305
27277	4.623822
32233	4.623738
49499	4.623643
44111	4.623057
41411	4.622722
15511	4.62204
55333	4.62094
72277	4.620785
33533	4.597536

Discussion

Interesting patterns overall persist across the different variations of Primel. Among the most interesting are the finding that the lowest two-digit and the lowest three-digit primes are the worst guesses for their respective variations and that the best five-digit Primel initial guess is almost exactly 10 times the best four-digit Primel initial guess.

The central practical result of this paper is as it pertains to playing Primel: when engaging in the *Lazy Strategy,* players should always start by guessing 82469.

References

Anderson, Benton J., and Jesse G. Meyer. "Finding the optimal human strategy for Wordle using maximum correct letter probabilities and reinforcement learning." *arXiv preprint arXiv:2202.00557* (2022).

de Silva, Nisansa. "Selecting Seed Words for Wordle using Character Statistics." *arXiv preprint arXiv:2202.03457* (2022).

Short, Martin B. "Winning Wordle Wisely." *arXiv preprint arXiv:2202.02148* (2022).

Primel archived link:
https://web.archive.org/web/20220211225932/https://converged.yt/Primel/